Open Problems in Mathematics, John F. Nash, and Riemann's Hypothesis

Talk dedicated to the memory of John F. Nash
(June 13, 1928 – May 23, 2015)

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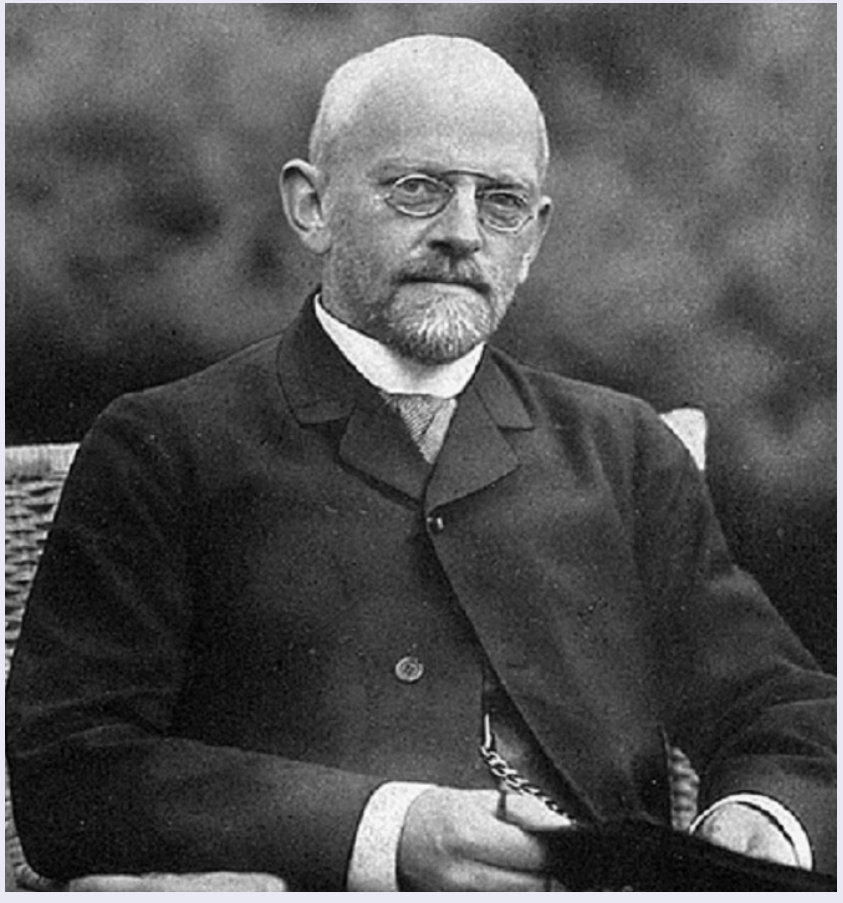
Hilbert's 23 Problems

From Hilbert's talk (ICM, Paris, 1900):

"Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries?"

David Hilbert



Open Problems in Mathematics

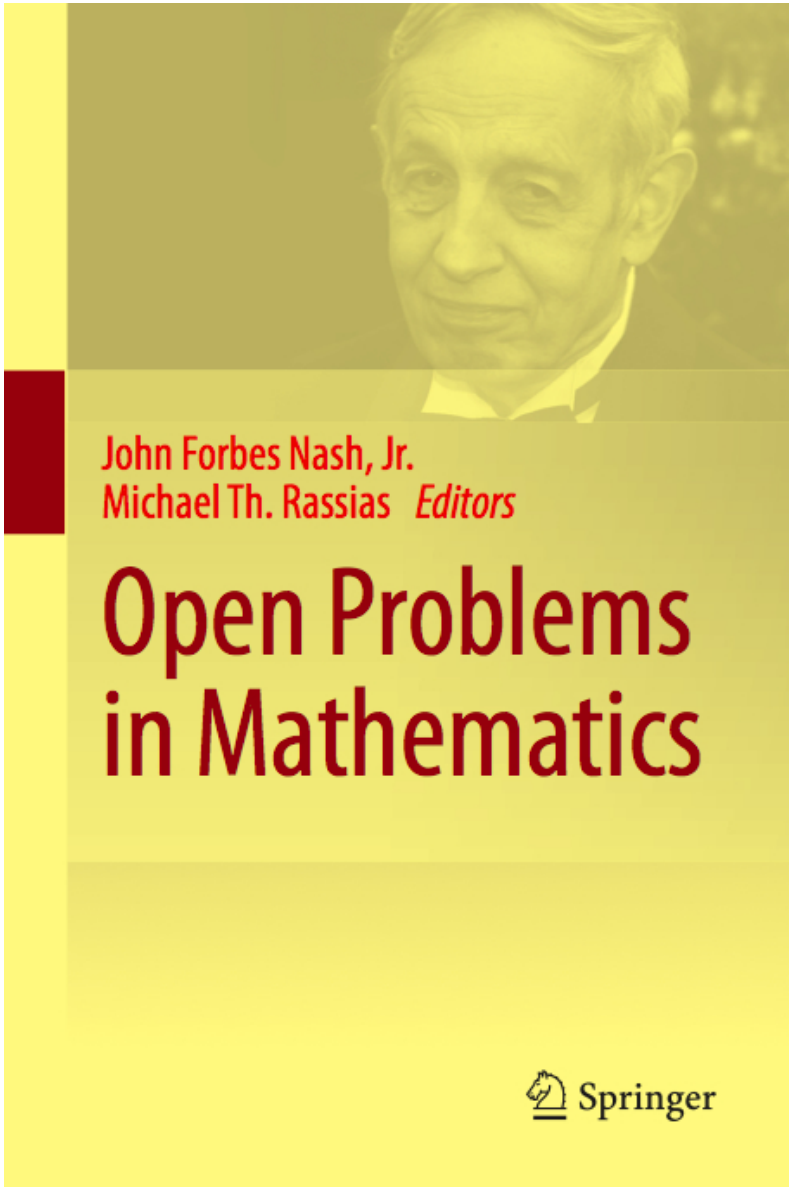
"It has become clear to the modern working mathematician that no single researcher, regardless of his knowledge, experience and talent, is capable anymore of overviewing the major open problems and trends of Mathematics in its entirety. The breadth and diversity of Mathematics during the last century has witnessed an unprecedented expansion. ... Perhaps Hilbert was among the last great mathematicians who could talk about Mathematics as a whole, presenting problems which covered most of its range at the time."

(From the Preface of the volume "Open Problems in Mathematics", J. F. Nash and M. Th. Rassias, Springer, 2016)

Modern trend:

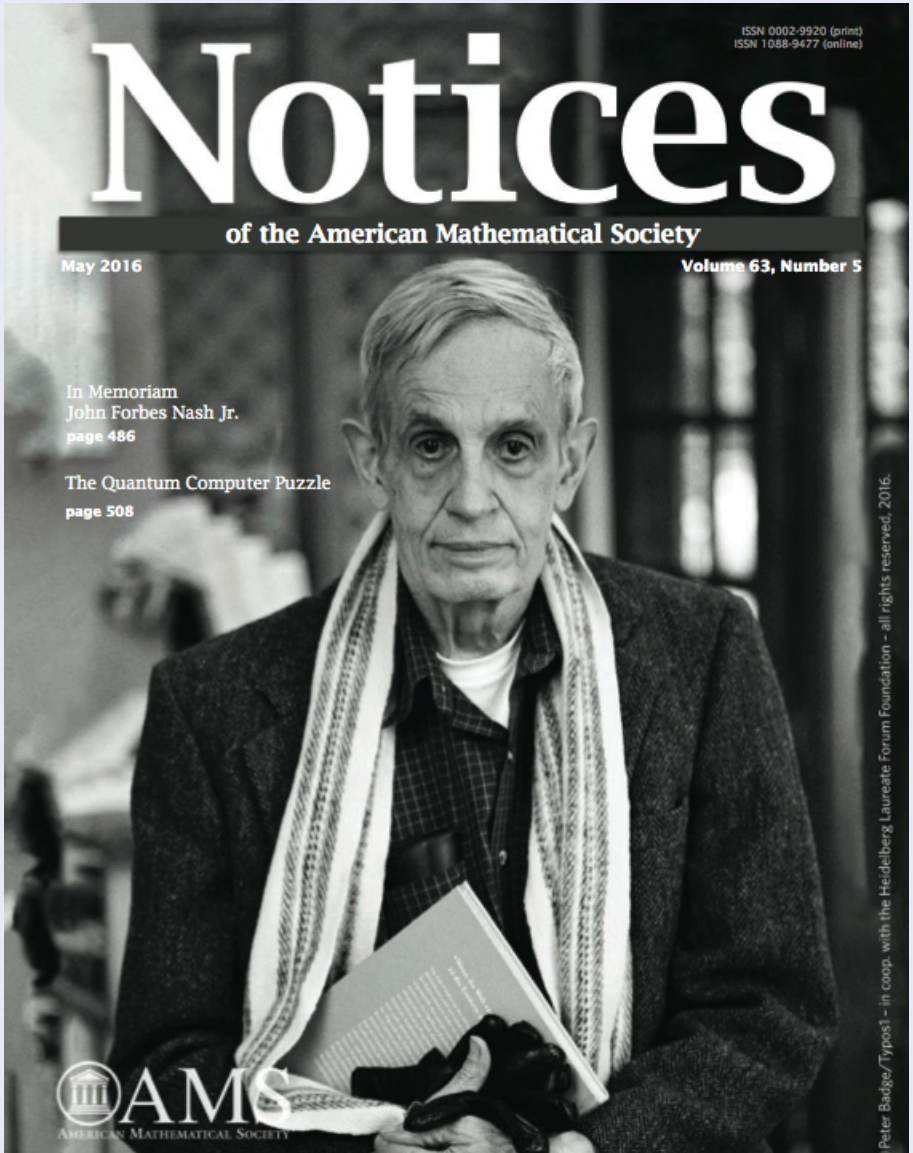
- Interdisciplinary methods in Mathematics and science as a whole
- From tunnels to bridges

Open Problems in Mathematics (cont.)



Open Problems in Mathematics (cont.)

Cover of Notices, AMS, May, 2016



Notices, AMS, May, 2016

Open Problems in Mathematics

Just before he left to collect his Abel Prize in Oslo in May 2015, Nash was working with Princeton postdoc Michael Th. Rassias to finish up the preface to an extraordinary book they edited together called *Open Problems in Mathematics*. The book will be published later this year by Springer.

The book consists of seventeen expository articles, written by outstanding researchers, on some of the central open problems in the field of mathematics today. Each article is devoted to one problem or a "constellation of related problems," the preface says. Nash and Rassias do not claim the book represents all of the most important problems in mathematics; rather, it is "a collection of beautiful mathematical questions which were chosen for a variety of reasons. Some were chosen for their undoubtable importance and applicability, others because they constitute intriguing curiosities which remain unexplained mysteries on the basis of current knowledge and techniques, and some for more emotional reasons. Additionally, the attribute of a problem having a somewhat *vintage flavor* was also influential in our decision process."

Here is another taste of the book, this one from the introduction, titled "John Nash: Theorems and Ideas" and written by Mikhail Gromov: "Nash was solving classical mathematical problems, difficult problems, something that nobody else was able to do, not even to imagine how to do it... But what Nash discovered in the course of his constructions of isometric embeddings is far from 'classical'—it is something that brings about a dramatic alteration of our understanding of the basic logic of analysis and differential geometry. Judging from the classical perspective, what Nash has achieved in his papers is as impossible as the story of his life... [H]is work on isometric immersions...opened a new world of mathematics that stretches in front of our eyes in yet unknown directions and still waits to be explored."

Nash and Rassias first met in September 2014 in the common room of the Princeton mathematics building, Fine Hall. Nash was eighty-six years old and probably the most famous mathematician in the world, and Rassias a twenty-seven-year-old Princeton postdoc who hails from Greece and had just finished his PhD at the ETH in Zurich. A chemistry developed between the two mathematicians and precipitated their collaboration on *Open Problems in Mathematics*. A Princeton News article that appeared on the occasion of Nash receiving the 2015 Abel Prize discussed Rassias's interactions with Nash (`www.princeton.edu/main/news/archive/S42/72/29C63/index.xml?section=topstories`). Rassias is quoted as saying: "Working with him is an astonishing experience—he thinks differently than most other mathematicians I've ever met. He's extremely brilliant and has all this experience. If you were a musician and had an opportunity to work with Beethoven and compose music with him, it'd be astonishing. It's the same thing."

Photo Princeton University, Office of Communications, Danielle Allen (2015).

Rassias talks to 2014 Abel Laureate Yakov Sinai as 2015 Abel Laureate Nash looks on.

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John Nash: Theorems and Ideas

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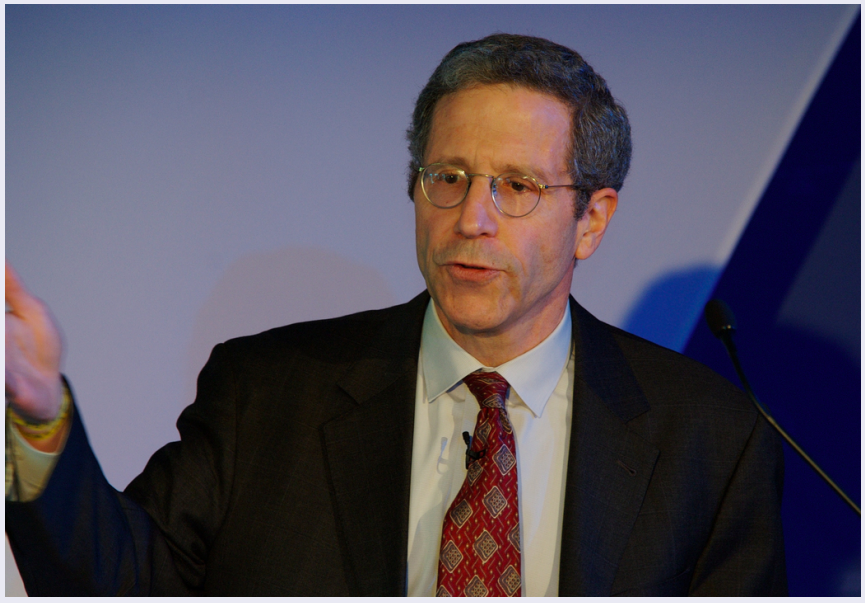
...what Nash has achieved in his papers [in analysis and differential geometry] is as impossible as the story of his life... [H]is work ...opened a new world of mathematics that stretches in front of our eyes in yet unknown directions and still waits to be explored.

Mikhail Gromov (Abel Prize, 2009)

Nash's last problem: A Mystery

John Nash was planning to write a brief article on an open problem in game theory. He was planning to prepare it and discuss about it after his trip to Oslo. Thus, he never got the opportunity to write it.
On this note, and notwithstanding the last minute invitation, Professor Eric Maskin generously accepted to contribute a paper presenting an important open problem in cooperative game theory.

Eric Maskin (Nobel Prize, 2007)